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LETTER TO THE EDITOR

Cross-over to global adiabatics in 2D ballistic transport

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Abstract. We introduce the concepts of *local* and *global* adiabatic regimes in quantum ballistic transport and discuss the cross-over between the two regimes. For a device consisting of micro-constrictions and wider 2D areas, local adiabatics leads to the absence of mode mixing at the constructions. Hence the conductance of a construction is quantised. Local adiabaticity is however not sufficient to ensure mode conservation in the entire device, i.e. to reach a global adiabatic regime. To achieve this a finite magnetic field has to be applied to the system. We have derived the necessary minimum strength of this field as a function of mode number and device geometry. As an example we discuss the manifestation of global adiabaticity in the Aharonov–Bohm effect.

Recently developed [1, 2] gate-controlled devices have for the first time made systematic studies of quantum transport in the ballistic regime possible. The key feature of the new devices based on GaAs heterostructures is the scale of the active area, which is sufficiently small compared to the length of the electron mean free path. This leads to collision-free, ballistic electron transport. A negatively biased split-gate structure produces depleted regions whose boundaries serve as the walls of a 'waveguide' for electron waves. The geometry of these walls is obviously of great significance for the purely quantum-mechanical transport process, and their smoothness is an essential feature. The small curvature of the boundaries on the scale of the Fermi wave-length, λ_F, of the two-dimensional electron gas (2 DEG), is due to the remoteness of the gates (producing depletion) from the 2 DEG. Introducing a single parameter, R, to characterise the boundaries by a radius of curvature, one has

$$R/\lambda_F \gg 1. \tag{1}$$

Under this condition, an electron wave propagating through a gate-controlled channel does not experience backscattering from boundary inhomogeneities. Moreover, in a sufficiently narrow part of the channel (figure 1), where

$$d(x) \ll R \tag{2}$$

the scattering between different transverse modes is suppressed. One is therefore able [3] to use an adiabatic approximation for describing the propagation of electron waves through a gate-induced constriction [1, 2]. For real gate geometries the condition (2) holds only on the scale

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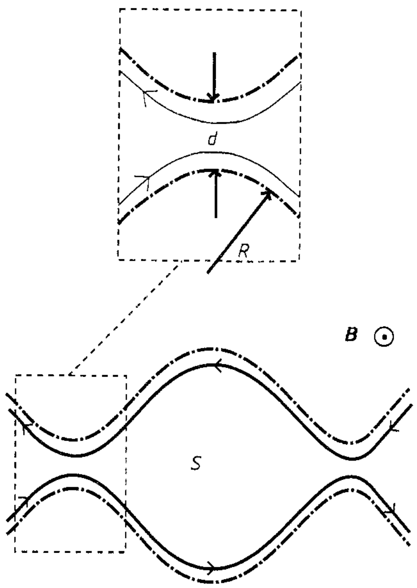

Figure 1. Boundaries of an electron 'waveguide' (chain curves) and typical edge states (full curves) for an electron in the presence of a magnetic field, $\boldsymbol{B}$. The largest curvature of the boundary is at the constriction (see enlargement). The system is in the *local* adiabatic regime if the radius of curvature, R, is large compared to width, d, of the constriction. The condition for *global* adiabaticity is discussed in the text.

$$x \ll R \tag{3}$$

where x is the distance from the narrowest part of the constriction. The adiabatic approach is therefore valid only *locally*. This is sufficient, however, as the conductance of a construction is determined by mode propagation on a scale [3] much smaller than R:

$$x \sim \surd(Rd(0)). \tag{4}$$

In the local adiabatic regime defined by equations (2)–(4) the geometry of a single micro-constriction determines its conductance, G, and causes [3] the step-like dependence on its width $G = G(d)$.

The adiabatic condition (2) breaks down at $x \sim R$ where even the small-angle scattering compatible with (1) mixes different modes. Experiments on devices with several constrictions serving as 'sources' and 'detectors' of electron modes show without any doubt evidence of mode-mixing (among various experiments we should especially mention those of [4] together with the discussion in [5]). Mixing of modes occurs not only in the absence of a magnetic field but also when a weak field is applied [6]. On the other hand, the anomalous quantum Hall effect observed in [7] demonstrates mode conservation over large distances, i.e. in an *entire* device. One concludes that a magnetic field may induce a cross-over from a *local* to a *global* adiabatic regime.

In this Letter we develop an adiabatic approach to electron propagation through edge states formed by the magnetic field near a curved boundary. We determine the magnetic field B_n^{global} necessary for cross-over to the global adiabatic regime and discuss the dependence of B_n^{global} on the radius of curvature, R, and the mode number n.

Electrons can be injected into the edge states of an active area through a construction. It is possible to prevent injection of a particular mode both by decreasing $d(0)$ and by increasing the magnetic field strength B. Hence the threshold valuye B_n of the field that switches the nth mode off (or on) depends on the value of $d(0)$. The possibility of tuning B_n by changing the gate voltage that controls $d(0)$ enables one to switch on the propagation of a new mode either in the mode-mixing ($B_n < B_n^{\text{global}}$) or in the adiabatic ($B_n > B_n^{\text{global}}$) regime, thus providing a tool for experimental study of the cross-over to the global adiabatic regime. An an example, we discuss the corresponding implications for the Aharonov–Bohm (AB) effect in a two-point contact device. A more detailed account of this work will be published elsewhere [8].

Scattering between different transverse modes corresponds to a redistribution of energy between the transverse and longitudinal motion of an electron. Consequently, it leads to a change in the wavevector, $k_n(x)$, of the longitudinal motion. The matrix elements mixing different modes are small if

$$|k_n(x) - k_{n\pm1}(x)|R \gg 1 \tag{5}$$

where R determines the length scale for spatial variations of the channel boundaries. Without a magnetic field the difference between $k_n(x)$ and $k_{n\pm1}(x)$ is *only* due to the finite width of the conducting channel and is of order $1/d(x)$. Hence (5) is violated for $x \gtrsim R$. In the presence of a magnetic field a new length scale, the radius of the cyclotron orbit, r_c enters the problem. When $r_c < d$, i.e. for sufficiently strong magnetic fields, the difference in wavevectors between two states of different modes at the Fermi surface will be of order $1/r_c$, hence finite even for a half-space where $d \to \infty$. This is due to the formation of edge states. In the presence of a magnetic field, the centre of the cyclotron orbit, y_n, for a system bounded in the y direction is an exact constant of motion [9] if the boundary is straight (x-independent). Each filled mode has an occupied edge state at the Fermi level, for which

$$k_n = (eB/c\hbar)\, y_n. \tag{6}$$

The energy of an eigenmode, E_n, depends on mode number, n, and on y_n, (see e.g. [10]). For states at the Fermi surface the position of the orbit centre is determined by the equation

$$E_n(y_n) = E_{\mathrm{F}}. \tag{7}$$

If the boundary differs from a straight line, e.g. has a step-like shape, y_n is no longer a good quantum number and inter-mode transitions occur. However, if the curvature, R^{-1}, that characterises this step is small enough, then even the most probable transitions between adjacent modes are suppressed. We shall now proceed to derive a more explicit criterion than (5) for adiabatic propagation in an edge state.

First we determine the adiabatic solutions to the Schrödinger equation in the presence of a magnetic field. Neglecting the small spin splitting, the orbital part of the Hamiltonian

$$\hat{H} = -(\hbar^2/2m)[\partial^2/\partial y^2 + (\partial/\partial x - \mathrm{i}\, eBy/c\hbar)^2]. \tag{8}$$

The adiabatic wavefunction that corresponds to the nth mode has the form

$$\Psi_n(x, y) = A_n(x)\varphi_n(x, y) \tag{9}$$

where $\varphi_n(x, y)$ satisfies the one-dimensional boundary value problem

$$\begin{aligned} &-(\hbar^2/2m)\{\partial^2/\partial y^2 - [k(x) - eB/c\hbar\, y]^2\}\, \varphi_n(x, y) = E_n\varphi_n(x, y) \\ &\varphi_n(x, y)|_{y=f(x)} = \varphi_n(x, y)|_{y\to\infty} = 0. \end{aligned} \tag{10}$$

The boundary conditions in (10) correspond to a constant electrostatic potential within the channel and an infinitely steep potential at the boundary, $y = f(x)$ (a 'hard' wall). We believe this to be a good approximation (see [8, 11, 12]. The eigenvalue E_n in (10) depends parametrically on $k(x)$ and $f(x)$. The function $k = k_n(x)$ for a given mode n is determined by an obvious analogue of (7).

The longitudinal part of the adiabatic wavefunction, $A_n(x)$ of (9), is

$$A_n(x) = \exp\left(\mathrm{i}\int^x k_n(x_1)\,\mathrm{d}x_1\right). \tag{11}$$

Because of the x-dependent boundary condition for φ_n in (10), the adiabatic wavefunctions Ψ_n of (9) are not exact solutions of the Schrödinger equation. However, we can use them as an orthogonal basis set for an expansion of the true wavefunction

$$\Psi(x, y) = \sum_n c_n(x) A_n(x) \varphi_n(x, y). \tag{12}$$

In this spirit of scattering theory, the wavefunction of (9) with a particular mode index n can be regarded as an incident wave propagating from the left ($x = -\infty$) where initially $c_l(x = -\infty) = \delta_{l,n}$ in (12). Then the set of coefficients $c_l(x = \infty)$ with $l \neq n$ determines the amplitudes a_{nl} for scattering to other modes. Inserting (12) into the Schrödinger equation and hence deriving a set of equations for c_n, one readily proves that the only sources for scattering are the spatial derivatives $\partial\varphi_n/\partial x$, and $\partial^2\varphi_n/\partial x^2$. One finds

$$\begin{aligned}
&-\frac{\partial^2 c_l}{\partial x^2} + 2\mathrm{i}\left[\frac{eB}{c\hbar} y_{ll}(x) - k_l(x)\right]\frac{\partial c_l}{\partial x} + \mathrm{i}\frac{\partial k_l}{\partial x} c_l \\
&\qquad + 2\mathrm{i}\frac{eB}{c\hbar}\sum_{m\neq 1} y_{lm}(x)\frac{\partial c_m}{\partial x}\exp\left(\mathrm{i}\int^x [k_m(x_1) - k_l(x_1)]\,\mathrm{d}x_1\right) \\
&\qquad = \int_{f(x)}^{\infty} \varphi_l(x, y)\left[-\frac{\partial^2\varphi_n(x, y)}{\partial x^2} + 2\mathrm{i}\left(\frac{eB}{c\hbar} y - k_n(x)\right)\frac{\partial\varphi_n(x, y)}{\partial x}\right]\mathrm{d}y \\
&\qquad \times \exp\left(\mathrm{i}\int^x [k_n(x_1) - k_l(x_1)]\,\mathrm{d}x_1\right).
\end{aligned} \tag{13}$$

The quantities $y_{lm}(x)$ in (13) are matrix elements of y with respect to the wavefunctions $\varphi_l(x, y)$. Because of the smoothness of the boundary, $y = f(x)$, on the scale of the Fermi wavelength, $\lambda_\mathrm{F} = 2\pi/k_\mathrm{F}$, only the second term on the left-hand side of (13) needs to be retained. The remaining first-order differential equation can easily be solved. The result for $l \neq n$ is

$$a_{nl} \equiv c_l(x \to \infty) = \int_{-\infty}^{\infty}\left[\alpha_{nl}\frac{\partial f(x)}{\partial x} + \beta_{nl}\left(\frac{\partial f(x)}{\partial x}\right)^2\right]\exp[\mathrm{i}(k_n - k_l)x]\,\mathrm{d}x. \tag{14}$$

Here $k_n - k_l$ is independent of x and the coefficients α_{nl} and β_{nl} are x-independent combinations of matrix elements involving φ_l [8]. The simple estimate $\alpha \simeq \beta \simeq k_\mathrm{F}$ is valid for almost all possible values of n. The spatial derivative $\partial f(x)/\partial x$ in (14) is dimensionless and depends on the shape of the boundary. The most natural way to characterise the boundary is by *single* parameter, R. The implication is that the variation is smooth but by no means small. As an illustration we shall evaluate (14) using a function that models a step-like boundary

$$\partial f(x)/\partial x = \exp[-(x/R)^2]. \tag{15}$$

The largest scattering amplitudes, a_{nl}, occur for scattering into adjacent modes,

$l = n \pm 1$. In the adiabatic regime of interest, these amplitudes should be small and one arrives at the following estimate for the scattering amplitudes

$$a_{n,n\pm1} \simeq \exp[\ln(k_F R) - \tfrac{1}{8}(eBR/c\hbar)^2 (y_n - y_{n\pm1})^2]. \tag{16}$$

We recall that the quantum numbers y_n in (16) are determined by (7) for a straight boundary. Now we use (7) and (16) to estimate the magnetic field, B_n^{global}, that determines the cross-over to the *global* adiabatic regime for the nth mode. Consider first skipping orbits in the weak magnetic field limit, where in the quasi-classical approximation

$$y_n = -[\![1 - \{[3\pi(n - \tfrac{1}{4})/4\sqrt{2}](B/B^*)\}^{2/3}]\!] r_c \qquad n = 1, 2, 3 \ldots \tag{17}$$

Here r_c denotes the cyclotron radius at the given field, B, and $B^* = c\hbar k_F^2/2e$ is a characteristic value of the magnetic field, related to the depopulation of the Landau levels. We determine the value of the field B_n^{global} that marks the cross-over to the global adiabatic regime ($B > B_n^{\text{global}}$) by requiring the argument of the exponential in (16) to be zero

$$B_n^{\text{global}}/B^* \simeq 0.33[\ln(2\pi R/\lambda_F)]^{3/4}(\lambda_F/R)^{3/2}\sqrt{n}. \tag{18}$$

It follows that the higher the mode number, n, the higher the magnetic field must be to reach the global adiabatic regime. Our assumption of a smoooth boundary guarantees that the factor $\sqrt{n}$ on the right-hand side of (18) is multiplied by a small coefficient. A typical value for the Fermi wavevector is [1, 2] $\lambda_F \simeq 400$ Å. The radius of curvature R can be estimated [13] as half the width of the lithographic gap of the split gate, $R \simeq 0.2\mu$ [1, 2]. Hence $B_n^{\text{global}}/B^* \simeq 0.1\sqrt{n}$. The mode number n in (18) should definitely be smaller than the full number of occupied Landau levels, around B^*/B. Consequently there exists a characteristic integer value N_R,

$$n_R = \text{Int}\{(R/\lambda_F)[0.23\ln(2\pi R/\lambda_F)]^{-1/2}\} \tag{19}$$

such that for all $n > n_R$ the threshold value necessary for global adiabatics is the same one

$$B^{\text{global}}/B^* \simeq [0.23\ln(2\pi R/\lambda_F)]^{1/2}(\lambda_F/R). \tag{20}$$

It is obvious that (18) and (20) match at $n = n_R$.

If $1/R$ is the largest curvature of the boundary in an active area, then a field larger than B^{global} of (20) is definitely sufficient to secure adiabatic transport in all edge states that exist in this area. The population of these states is therefore not changed as electrons propagate through the active region and is determined only by the source injecting the carriers. The advantage of gate-controlled devices is that the gate bias defines both the active area and the point contacts serving as injectors. These point contacts have the form of micro-channels connecting the active area with the 2D leads. The tunable character of the contacts and the *local* adiabatic conditions in their vicinity [3] makes it possible to control the number of modes in the active region that are in contact with the leads. For a sufficiently narrow channel this number obviously can be made smaller than n_R given by (19). Under these conditions even a field smaller than B^{global} of (20) will be sufficient to maintain *global* adiabatic conditions in the whole active area. Actually for global adiabaticity (18) needs to be satisfied for the propagating mode with the highest mode number (due to the monotonic dependence of B_n^{global} on n in (18)). On the other hand the number of propagating modes can be restricted both by shrinking the width, $d = d(0)$, of the micro-channel and by applying a magnetic field, $\boldsymbol{B}$. So, to determine the set of parameters, $(d, \boldsymbol{B})$, where global adiabaticity holds for a device of the type shown in figure 1, we have to determine the switching-off condition for a mode.

The boundaries of a channel lift the degeneracy of the Landau levels produced by a magnetic field and broaden each of them into a band. For boundaries of sufficiently large radius of curvature, $R \gg d$, the threshold value, B_n, when the constriction becomes opaque to all states in a band, can be determined [13] from the condition

$$E_n(\boldsymbol{B}, d) = E_F. \tag{21}$$

Here $E_n(\boldsymbol{B}, d)$ is the minimum energy in the nth band for an electron in a condition channel of constant width d. To determine the bands one can use (10) with a constant wavevector $\boldsymbol{k}$ and boundary conditions $\varphi_n(y = 0) = \varphi_n(y = d) = 0$. Depending on the ratio $2r_c/d$ the band energies are determined mostly by either spatial or magnetic quantisation. For a wide channel (or a strong magnetic field), where $d/2 > r_c$, the boundaries change the energy of a Landau level only slightly. This is because only the exponentially small 'tails' of the wavefunction in the classically forbidden regions are affected. Treating them in the WKB approximation one finds small corrections to the bulk energy caused by the boundaries. After straightforward calculations [8] starting from (21) one finds for a mode-separating line, $B_n(z)$, the following parametrical description:

$$B^*/B_n + \tfrac{1}{2} = n + (1/\pi)\exp[-(2n - 1)g(u)] \tag{22}$$

where the function g is

$$g = u\sqrt{u^2 - 1} - \ln(u + \sqrt{u^2 - 1}) \qquad u = (\pi/4)[z/(n - \tfrac{1}{2})]. \tag{23}$$

In (22) and (23) we have used the convenient dimensionless units $z = k_F d/\pi$ and B^*/B related to the experimentally controlled parameters d and B.

In the opposite limit of a narrow channel (or a weak magnetic field), $d/2 < r_c$, the eigenvalues E_n are determined mainly by spatial quantisation. The magnetic field merely adds small corrections which can be calculated by standard perturbation theory [13]. One finds

$$(n/z)^2 + \tfrac{1}{48}\pi^2(B_n/B^*)^2 z^2 = 1. \tag{24}$$

Equation (24) indicates that the mode-separating lines in the $(B^*/B, z)$ plane asymptotically tend to $z = n$ as $B \to 0$, i.e. $B^*/B \to \infty$.

The results in the two limiting cases discussed above match quite well in the intermediate region, $d/2 \simeq r_c$. Mode-separating lines in the dimensionless coordinates B^*/B and z are plotted in figure 2. Using (18) and (20) we have also marked the crossover field B_n^{global} by the broken curve in figure 2. From the figure it is obvious that depending on the particular value of z, i.e. gate voltage, we can switch on a new mode by varying B either in the mode-mixing regime (to the right of the broken line in figure 2) or in the global adiabatic regime.

The two regimes mentioned above can be directly distinguished in an experiment with a device of the shape [14] sketched in figure 1. For particular values of $(z, B^*/B)$ close to the mode-separating lines an oscillatory pattern emerges in the field-dependent conductance $G(B)$ due to the Aharonov–Bohm (AB) effect. If global adiabaticity holds then only a *single* mode contributes to the oscillations which consequently have a well defined period. In contrast to the original AB effect the electrons are moving in a region of finite magnetic field. Therefore the magnetic flux enclosed by the electron orbit is determined by the field not only directly in the usual fashion but also indirectly via the

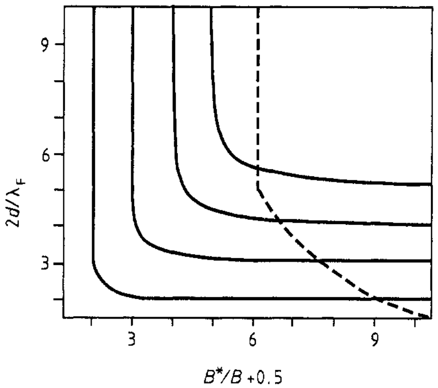

Figure 2. Mode-separating lines for modes $n = 2$–5. The broken curve marks the cross-over from mixed-mode propagation in weak fields (above and to the right of the broken line) to a global adiabatic regime without inter-mode scattering in relatively strong fields (below and to the left of the broken line). The curve was plotted for the realistic parameters $R = 0.2\ \mu m$, $\lambda_F = 400$ Å but is *not* universal. This gives experimental freedom to shift it relative to the mode-separating lines which are universal when plotted in the dimensionless variable used here.

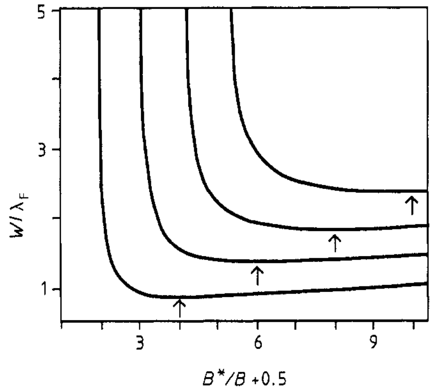

Figure 3. Corrections to the 'geometrical' value for the period of Aharonov–Bohm oscillations, ΔB, for modes $n = 2$–5. The present result, $\Delta B = (2\pi\hbar/c)[S - LW(B)]^{-1}$ depends on both n and B. For each mode n the function $W = W_n(B)$ has a minimum (marked by an arrow).

field dependence of the area, S_{eff}, encircled by the orbit. The AB period can be expressed as

$$\Delta B = (2\pi c\hbar/e)[\partial(BS_{\text{eff}}/\partial B]^{-1} \qquad S_{\text{eff}} = S - L(y_n + r_c). \tag{25}$$

Here S and L are the area and perimeter length of the cavity in figure 1. Equation (25) contains deviations from the geometrically determined period, $2\pi c\hbar/eS$. These deviations can be described by a quantity $W_n(B)$ of dimension length: $\Delta B = (2\pi c\hbar/e)[S - LW_n(B)]^{-1}$. For the calculation of $W_n(B)$ we can again use the WKB approximation (see [8] for details). Due to the distinct features of $W_n(B)$ displayed in figure 3 the dependence of ΔB on B can be used as an indication of global adiabatic transport.

The field-induced cross-over to the global adiabatic regime has implications also for several other phenomena. Here we shall only mention the anomolous quantum Hall effect [7] and the suppression of Shubnikov–de Haas oscillations [6].

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References

[1] van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhoven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848

[2] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A C 1988 *J. Phys. C: Solid State Phys.* **21** L209

[3] Glazman L I, Lesovik G B, Khmel'nitskii D E and Shekhter R I 1988 *JETP Lett.* **48** 239

[4] Wharam D A, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones J A C 1988 *J. Phys. C: Solid State Phys.* **21** L887

[5] Beenakker C W J and van Houten H 1989 *Phys. Rev.* B at press

[6] van Wees B J, Willems E M M, Kouwenhoven L P, Harmans C J P M, Williamson J G, Foxon C T and Harris J J 1989 preprint

[7] van Wees B J, Willems E M M, Harmans C J P M, Beenakker C W J, van Houten H, Williamson J G, Foxon C T and Harris J J 1989 *Phys. Rev. Lett.* **62** 1181

[8] Glazman L I and Jonson M 1989 *Phys. Rev.* B submitted

[9] Landau L L and Lifshitz E M 1958 *Quantum Mechanics* (London: Pergamon)

[10] Beenakker C W J, van Houten H and van Wees B J 1989 *Festkörperprobleme/Advances in Solid State Physics* vol 29 ed. U Rössler (Braunschweig: Pergamon/Vieweg)

[11] Laux S E, Frank D J and Stern F 1988 *Surf. Sci.* **196** 101

[12] Wharam D A, Ekenberg U, Pepper M, Hasko D G, Ahmed H, Frost J E F, Ritchie D A, Jones D C and Jones G A C 1989 *Phys. Rev.* B **39** 6283

[13] Glazman L I and Khaetskii A V 1989 *J. Phys.: Condens. Matter* **1** 5005–11

[14] van Wees B J, Kouwenhoven L P, Harmans C J P M, Williamson J G, Timmering C E, Broekaart M E I, Foxon C T and Harris J J 1989 *Phys. Rev. Lett.* **62** 2523